COMPLEX NUMBERS AND DIFFERENTIAL EQUATIONS

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SYNOPSIS

I. COMPLEX NUMBERS

A. GETTING STARTED
1. Definitions, Cartesian representation
2. Argand diagram
3. Polar form
4. Complex exponentials
5. Arithmetic manipulation
6. Curves in the complex plane

B. DE MOIVRE'S THEOREM
1. De Moivre's theorem
2. Trig. functions of multiple angles $\rightarrow$ powers of trig. functions
3. Powers of trig. functions $\rightarrow$ trig. functions of multiple angles
4. Powers and roots of complex numbers
5. Polynomials: sums and products of roots
6. Using complex numbers and the roots formulas to prove trig. identities

C. OTHER APPLICATIONS OF COMPLEX NUMBERS
1. Summing trig. series
2. Integration

D. FUNCTIONS OF A COMPLEX VARIABLE
1. Exponentials
2. Logarithms
3. Trig. and hyperbolic
4. Inverse trig. and hyperbolic

II. FIRST ORDER DIFFERENTIAL EQUATIONS

0. Terminology
1. Separable
1′. Almost separable
2. Homogeneous
2′. Homogeneous but for constant
2″. Looks like 'homogeneous but for constant' but is 'almost separable' 3. Integrating factor
4. The Bernoulli equation
5. Exact equations
6. Oddments

III. SECOND ORDER DIFFERENTIAL EQUATIONS

0. More terminology and the principle of superposition
1. Second order, linear, homogeneous DEs with constant coefficients:
auxillary equation has real roots
auxillary equation has complex roots
auxillary equation has repeated roots
2. The damped oscillator
3. Second order, linear, inhomogeneous DEs with constant coefficients:
finding the particular integral
4. Oddments
a. Euler's equation
b. Integration wrt the dependent variable
c. dependent variable 'missing'

IV. FORCED OSCILLATORS AND RESONANCE

1. The forced oscillator
2. Transient solution
3. Steady state solution
4. The amplitude response
5. Width of resonance and the Q-factor
6. Power and energy
7. Phase
8. The LCR circuit

V. COUPLED DIFFERENTIAL EQUATIONS